MELTING THE MANTLE

1. MOTIVATION

IMPORTANT TO UNDERSTAND HOW + WHY MELTING OCCURS
- MELTING $\rightarrow$ CRUST FORMATION, PLANETARY DIFFERENTIATION
- MELTING $\rightarrow$ DEGASSING $\rightarrow$ ATMOSPHERE
- MELTING $\rightarrow$ REDISTRIBUTION OF TRACE ELEMENTS

MELTS CAN GIVE INSIGHT INTO
- THERMAL STATE OF EARTH'S INTERIOR
- COMPOSITION OF EARTH'S INTERIOR

KEY QUANTITIES WE WILL WANT ARE

T - temperature of melting

P - of melting

F - degree of melting (fraction of melting)

2. WHAT IS MELTING?

PHASE CHANGE FROM SOLID TO LIQUID

WHAT IS A SOLID... LIQUID?

- SOLID MINERAL PHASE HAS A CRYSTALLINE STRUCTURE
  LONG RANGE ORDER

LO ENTROPY
e.g. SiO_2, WHERE ALL SiO_4^{4-} ARE BONDED

- LIQUID - SHORT RANGE ORDER

HI ENTROPY
e.g. MOLTEN SiO_2; SOME bonds are broken

* BE CAREFUL NOT TO CONFUSE THE DEFINITION OF
A LIQUID WITH A RHEOLOGICAL FLUID.

3. MELTING OF A 1-COMPONENT (e.g. pure) SYSTEM

ENERGY NEED TO MELT
$\hookrightarrow$ HEAT OF FUSION ΔH_f

○ THIS HEAT IS USED TO BREAK OR STRETCH BONDS
$\hookrightarrow$ ENTROPY INCREASES

$$\Delta S = \frac{\Delta H_f}{T_m}$$

EXAMPLE,

$$SiO_{2 \, qtz} = SiO_{2 \, melt}$$

$\boxed{P \text{ constant}}$

T | S | S+L | L

$\int dH = \int C_p dT$

T - CONSTANT

$\int dH$ = cumulative heat added

1.0

F (melt fraction)

0.0

$\int dH$

S

$\int dH$

SOLIDUS
F=0

LIQUIDUS
F=1
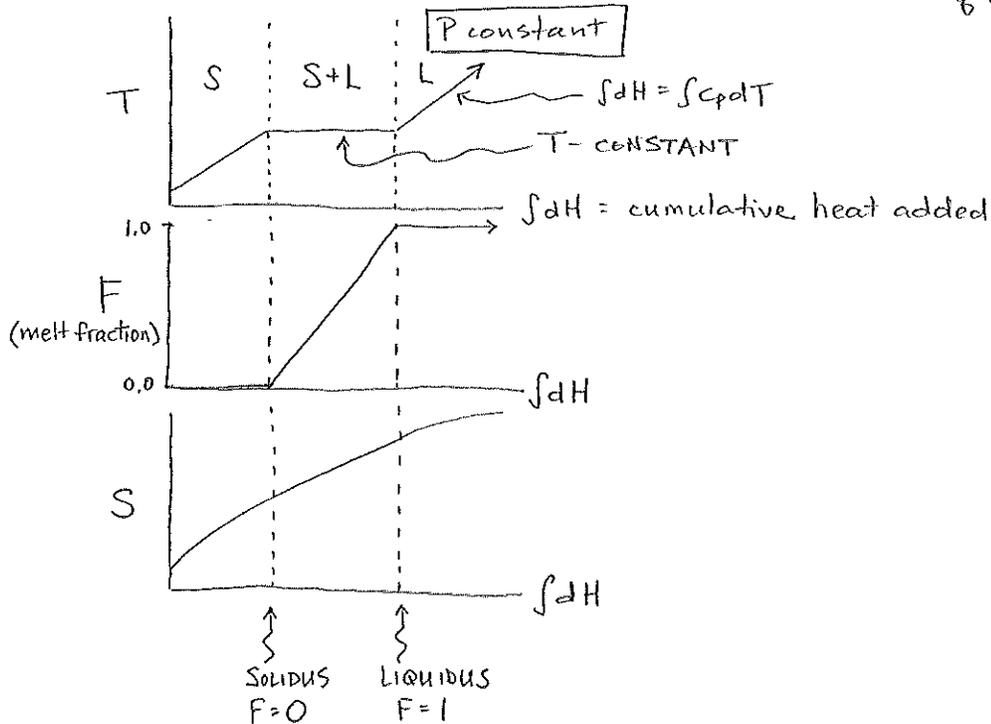
* MELTING IS A PHASE CHANGE

* IN 1-COMPONENT SYSTEM, 2 PHASES COEXIST DURING MELTING AT CONSTANT P, T CANNOT CHANGE UNTIL ONE PHASE IS CONSUMED.

2

MELTING PHASE DIAGRAM IN P-T SPACE

- 1 COMPONENT SYSTEM

$$SiO_2\text{ }_{solid} = SiO_2\text{ }_{Liquid}$$
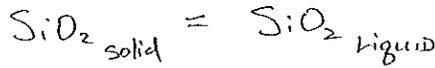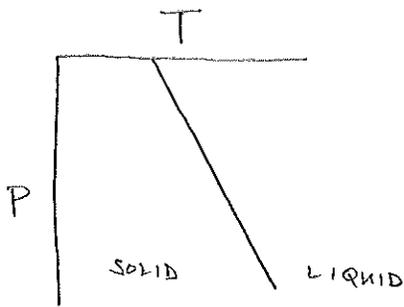
$$\Delta V_{rxn} = V_{Liq} - V_{solid} > 0$$

$$\Delta S_{rxn} = S_{Liq} - S_{solid} > 0$$

SEE BELOW ↓

$$\left(\frac{dP}{dT}\right)_{rxn} = \frac{\Delta S_{rxn}}{\Delta V_{rxn}} > 0$$

↳ CLAPEYRON SLOPE

IN GENERAL $\frac{dP}{dT} > 0$ FOR MELTING RXNS

NOTE $\left(\frac{dP}{dT}\right)_{rxn} = \frac{\Delta H_f}{T_f \Delta V_f}$ ⤒fusion

IF > 0, THEN ENDOTHERMIC (ASSUMING $\Delta V_f > 0$)

→ MELTING CAN OCCUR BY INCREASE IN T, DECREASE P

isoBARic MELTING

DECOMPR. MELTING S \ L
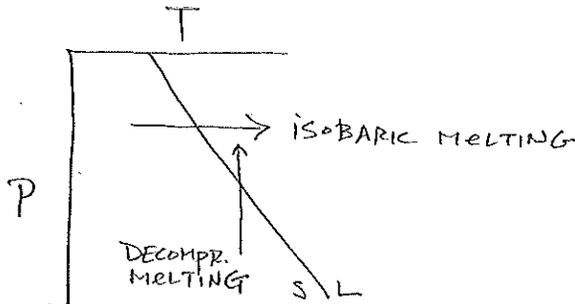
*NOTE: WITH INC. P

$$\Delta V_{melting} \longrightarrow 0$$

$\frac{dP}{dT}$ becomes large

* DERIVATION OF CLAPEYRON SLOPE

1st LAW

internal energy = heat + work

$dU = q + w$

$dU = TdS - pdV$

$U(S,V) = TS - PV$

DEFINE G (USING LEGENDRE TRANSF.)

$G \equiv U - TS + PV = H - TS$

$G \equiv H - TS$ ← entropy

Gibbs Free Energy   enthalpy (heat + bond energy)

$dG = dU - TdS - SdT + VdP - VdP$

combine w/ $dU = TdS - pdV$

$dG = -SdT + VdP$

FOR A RXN $d\Delta G_{rxn} = -\Delta S_{rxn}dT + \Delta V_{rxn}dP$

$d\Delta G_{rxn} = 0$ at equilibrium $\Rightarrow \frac{dP}{dT} = \frac{\Delta S_{rxn}}{\Delta V_{rxn}}$

3

DECOMPRESSION MELTING IS IMPORTANT IN EARTH

ΔP large, ΔT small

HOW DOES DECOMPRESSION MELTING OCCUR?

※ CONSIDER A HOT BLOB IN THE MANTLE.

$\uparrow V$

ΔT
ρ_0

Radius R
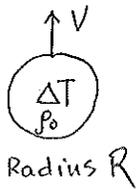
$\Delta\rho = \rho_0 \alpha \Delta T$ thermal expansion $\rightarrow$ density decrease

IF BUOYANT FORCE EXCEEDS VISCOUS RESISTANCE...

UPWARDS VELOCITY $V \sim \dfrac{\Delta\rho g R^2}{\eta}$

BLOB WANTS TO COOL

COOLING TIME $\sim \dfrac{R^2}{K}$

CHARACTERISTIC TRANSPORT TIME $\sim \dfrac{R}{V}$

WHEN $\dfrac{\text{COOLING } t}{\text{ASCENT } t} \gg 1$, RISES FASTER THAN IT COOLS

$$\frac{\Delta\rho g R^3}{\eta K} = \frac{\rho_0 \alpha g \Delta T R^3}{\eta K} = \text{Rayleigh number}$$

∴ $Ra > Ra_{cr} \sim 10^3$ CONVECTION

EARTH $Ra \sim 10^5 - 10^7$

CONCLUSION $\rightarrow$ CONVECTION LEADS TO ADIABATIC UPWELLING

e.g. $q = 0$

$\uparrow V$

no heat loss

T

P

S^L

4
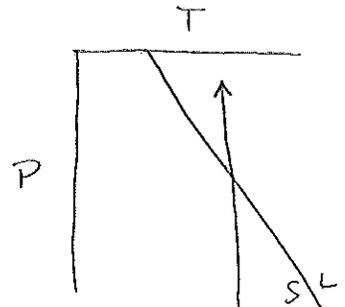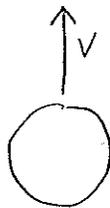
ADIABATIC DECOMPRESSION

$q = 0$

since $TdS \equiv q$, for a reversible process $\rightarrow dS = 0$

ISENTROPIC PROCESS

$$\Delta U = q + W$$

$$\Delta U = TdS - pdV$$

isentropic

$$\Delta U = 0 - pdV$$

Isentropic decompression causes upwelling MANTLE TO DO WORK ON SURROUNDINGS, SO INTERNAL ENERGY DROPS (AND T drops)

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{\alpha T}{\rho C_p} \qquad OR \quad \left(\frac{\partial T}{\partial z}\right)_S = \frac{\alpha g T}{C_p}$$

SOLID $\left(\frac{\partial T}{\partial z}\right)_S \sim 0.5°C/km$

Compare with SOLIDUS $\sim 3-4 °C/km$ (NOTE THIS VARIES WITH DEPTH)
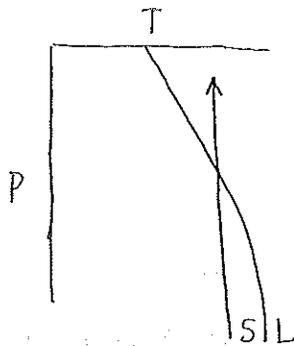
WHAT DOES ISENTROPIC MELTING LOOK LIKE IN A
   1 - COMPONENT SYSTEM

T

F = 1, LIQUIDUS

F = 0, SOLIDUS

$\frac{\Delta H_f}{C_p} = \Delta T$

S \ L

P
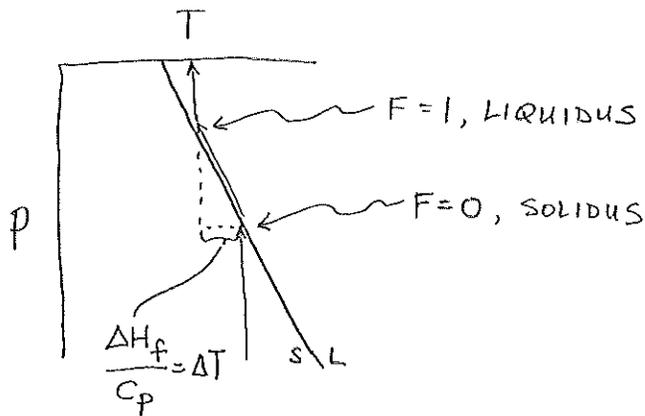
* AT CONSTANT P,
    $T_{SOLIDUS} = T_{LIQUIDUS}$

* BUT NOT SO, DURING
    DECOMPRESSION

T DROPS DURING MELTING BECAUSE HEAT ABSORBED INTO
PHASE CHANGE

T T_p = POTENTIAL TEMPERATURE

MELTING ADIABAT

S \ L

P

SOLID ADIABAT
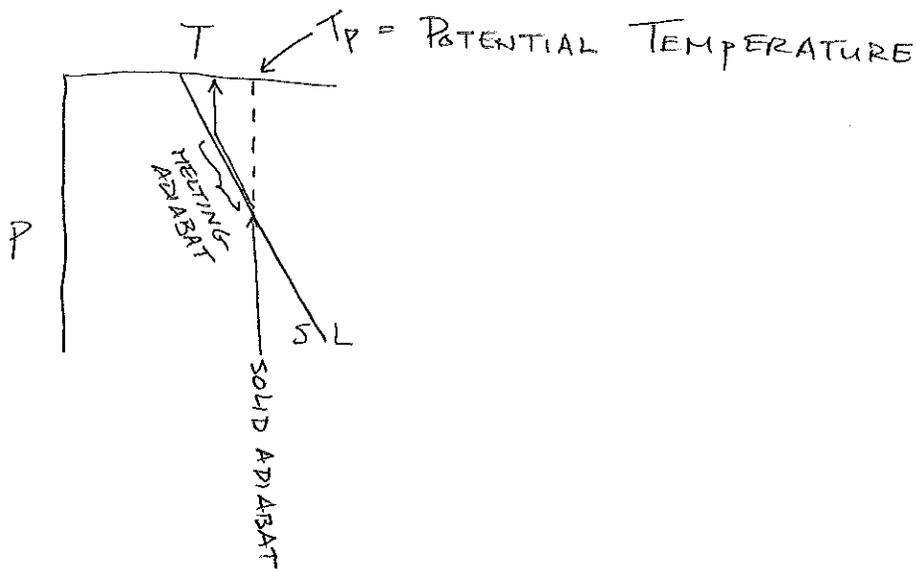
MELTING IN MULTI-COMPONENT SYSTEM
BINARY EXAMPLE

$$\overline{G}_{T,P} = \sum X_i \mu_i \quad \longleftarrow \text{chemical potential}$$

↑ molar Gibbs Free Energy of phase

↙ mole fraction of component

$$\mu_i = \mu_i^o + RT \ln X_i \quad \text{(assume ideal mixing)}$$

$$\overline{G}_{T,P} = \sum X_i \mu_i^o + RT \sum X_i \ln X_i$$

note similarity to $\Delta S_{mixing} = \sum R X_i \ln X_i$

$$RT \sum X_i \ln X_i = TS$$
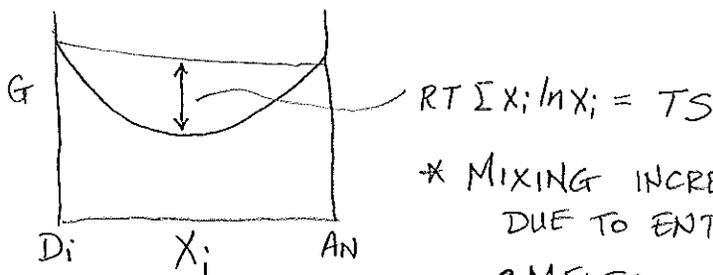
* MIXING INCREASES STABILITY OF A PHASE
DUE TO ENTROPY INCREASE $\rightarrow$ G DECREASE

° MELTS = SOLUTIONS

° MINERALS = IF SOLID SOLUTION, e.g. $(Mg,Fe)_2SiO_4$

P = CONSTANT

L

LIQUIDUS

Di+L   AN+L   T

T_L

$T_E \rightarrow$ SOLIDUS

Di + AN

X_E

ANALOG OF A BASALT

Di X_B X   AN
$CaMgSi_2O_6$ $CaAl_2Si_2O_8$
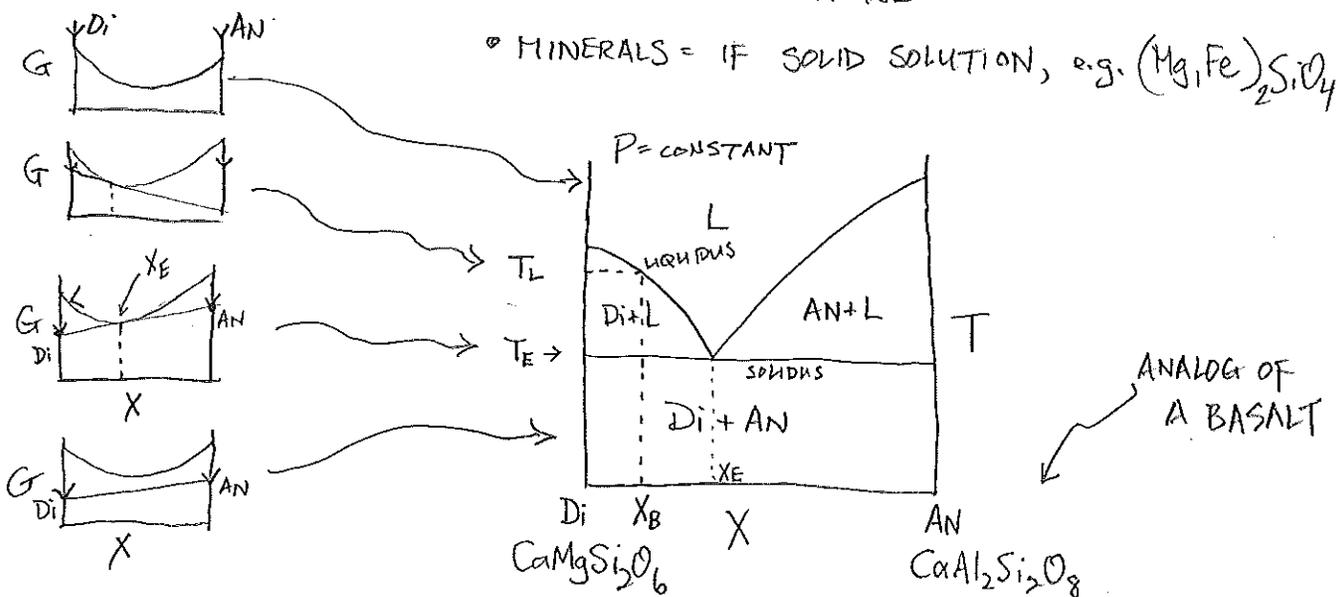
RULES OF THUMB

° ALL LIQUIDS AT THE ONSET OF MELTING HAVE SAME COMPOSITION
(IN THIS, EUTECTIC COMP., X_E)

° $T_E = T_{SOLIDUS}$

° $T_{LIQUIDUS} \neq T_{SOLIDUS}$

BINARY XMPL, EFFECT OF P

$$\left(\frac{\partial G}{\partial P}\right)_T = V$$

B (hi V, lo ρ)

A (Lo V, hi ρ)

G

P

$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

G

Lo S solid

hi S melt

P
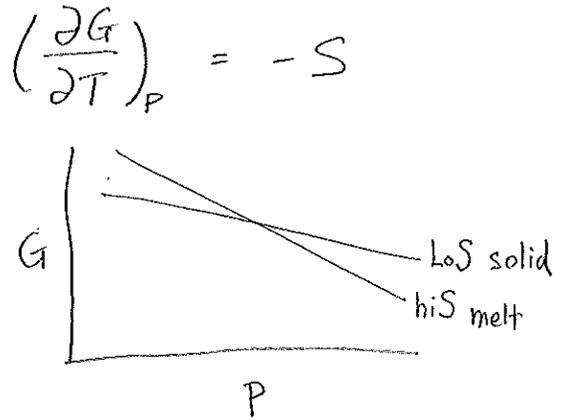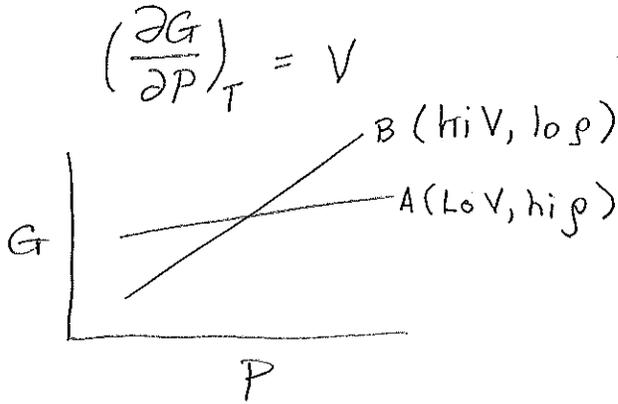
RULES OF THUMB

* Hi P favors THE STABILITY of <u>DENSE</u> PHASES

∴ HARDER TO MELT AT HIGHER P, DO T_E RISES

∴ X_E, EUTECTIC COMPOSITION, MOVES AWAY FROM Hi ρ PHASE
(i.e. phase field for hi ρ phase expands

* Hi T favors hi entropy phases
melts require hi T.

BASALT ANALOG

--- 1 GPa
— 1 atm

T

Di

$X_E \rightarrow$

AN
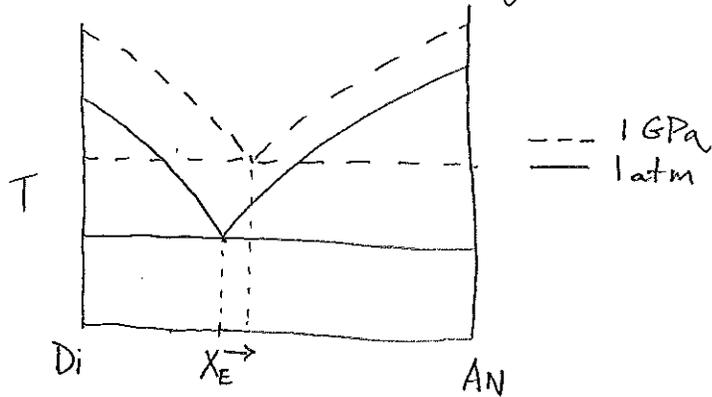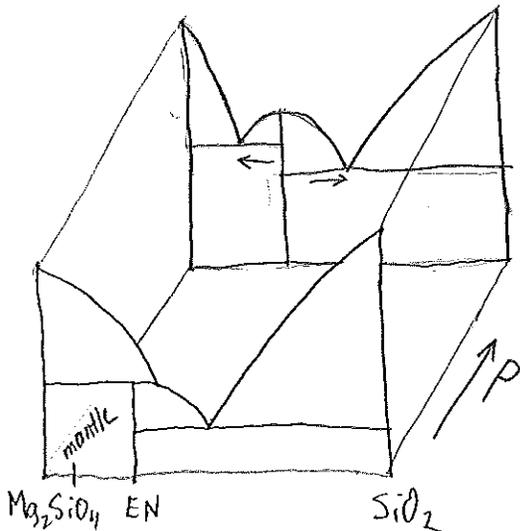
MANTLE PERIDOTITE ANALOG
Mg_2SiO_4 OLIVINE
ENSTATITE $Mg_2Si_2O_6$

P ↑ SiO_2 in melt ↓

mantle

Mg_2SiO_4 EN

SiO_2

$\nearrow P$

SiO_2 molt

P

8
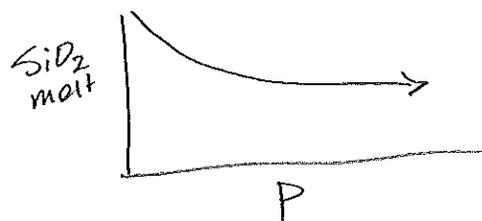
ISENTROPIC DECOMPRESSION MELTING IN MULTI-COMP. SPACE
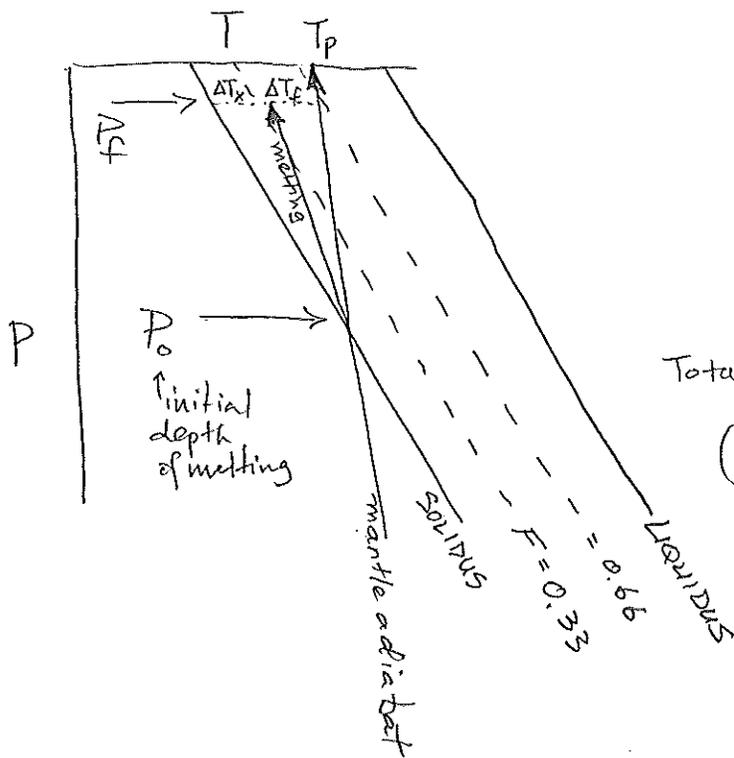
$$\Delta T_f = \frac{F \Delta H_f}{C_p}$$

$$\Delta T_x = F \frac{dT}{F}$$

Total heat content

$$\left(P_o - P_f \right) \left(\frac{\partial T}{\partial P}\right)_{solidus} - \left(P_o - P_f\right)\left(\frac{\partial T}{\partial P}\right)_{adiab}$$

MELTING AT A RIDGE
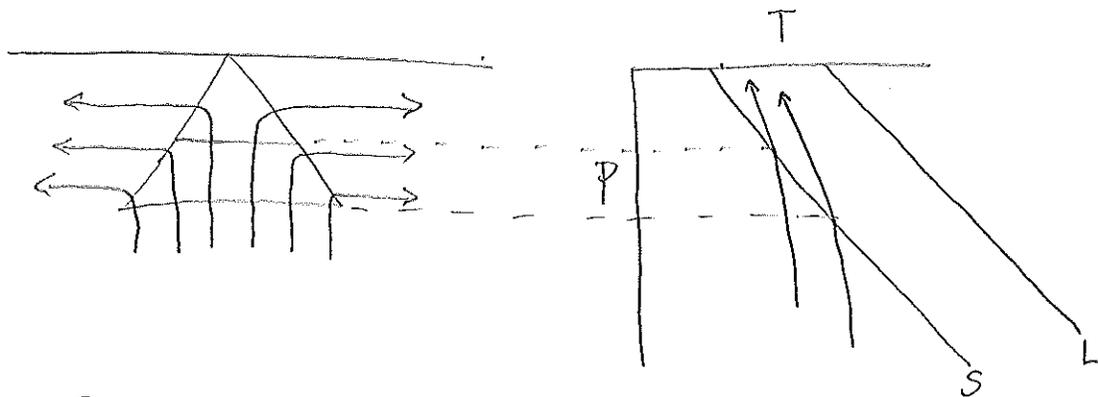
RULES OF THUMB

hi T_p $\Rightarrow$ hi F , hi P_o

Lo T_p $\Rightarrow$ lo F , lo P_o

IF THERE IS A LITHOSPHERIC CAP
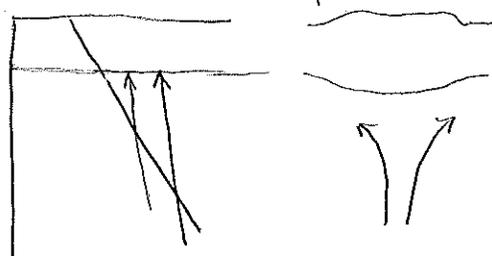
$$(P_f - P_o) \text{ small} \Rightarrow \text{lo } F$$

EFFECT OF VOLATILES

SOLIDUS IS DEPRESSED $\underline{\underline{IF}}$ VOLATILES ARE SOLUBLE IN MELT.

H_2O, CO_2 SOLUBLE IN SILICATE MELT

CH_4, C $\underline{NOT}$ SOLUBLE SO REDOX MATTERS

BECAUSE SOLUBILITY VARIES AS P, FREEZING POINT DEPRESSION IS SMALLER AT LO P.
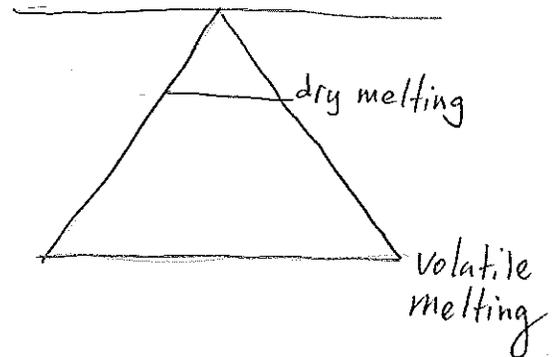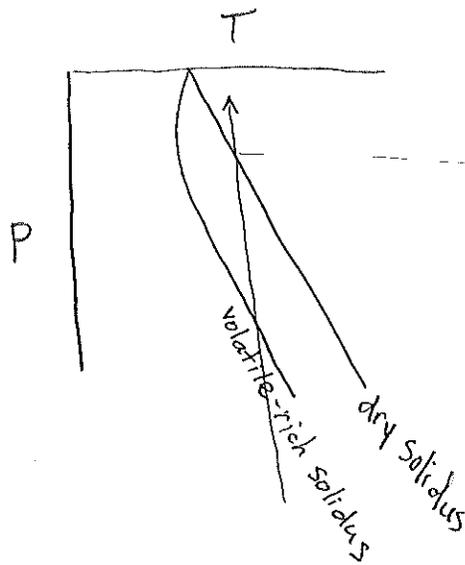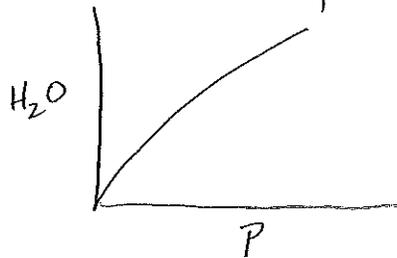
GOAL IS TO GET T_p BECAUSE THIS GIVES THERMAL STATE
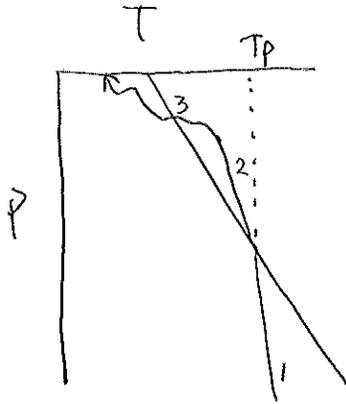
1- solid adiabat

2- melting adiabat

3- non-adiabatic cooling

IF WE KNOW T, F OR P of PRIMARY MAGMA THEN WE CAN SAY SOMETHING ABOUT T_p.

PRIMARY = IN EQUILIBRIUM W/ MANTLE SOURCE

EVOLVED = NOT IN EQ W/ SOURCE DUE TO FRACTIONAL CRYSTALLIZATION OR CONTAMINATION

HOW DO WE GET T_p?

F, T, P of MELTING?

F CAN BE DETERMINED FROM INCOMPATIBLE TRACE ELEMENTS

SIMPLE APPROACH —
BATCH EQUILIBRIUM MELTING
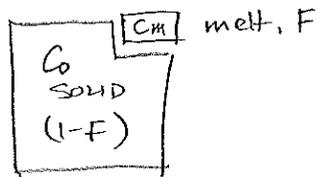

PARTITION COEFFICIENT

$$D = \frac{C_{SOLID}}{C_{MELT}}$$

>1 COMPATIBLE

<1 INCOMPATIBLE

$$C_0 = C_m F + C_s (1-F)$$

$$\boxed{\frac{C_m}{C_0} = \frac{1}{D + F(1-D)}}$$ ← melt enrichment factor
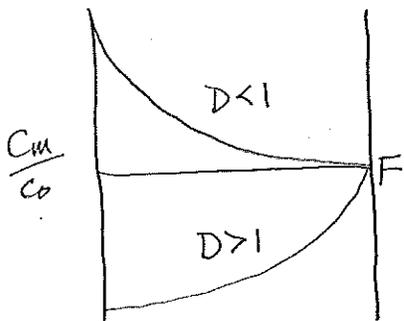
general rules: IF $D = 0$ $\quad \frac{C_m}{C_0} = \frac{1}{F}$

IF $F = 0$ $\quad \frac{C_m}{C_0} = \frac{1}{D}$

11

<u>application</u>

T_i | × × × ×$_×$ ×$_×^×$ ×$_×$ × × ×   Lo F melt (assumes C_0 constant)

⟵ olivine fractionation ○ ○ ○ ○ ○ ○ ○ ○ ○   hi F melt

MgO
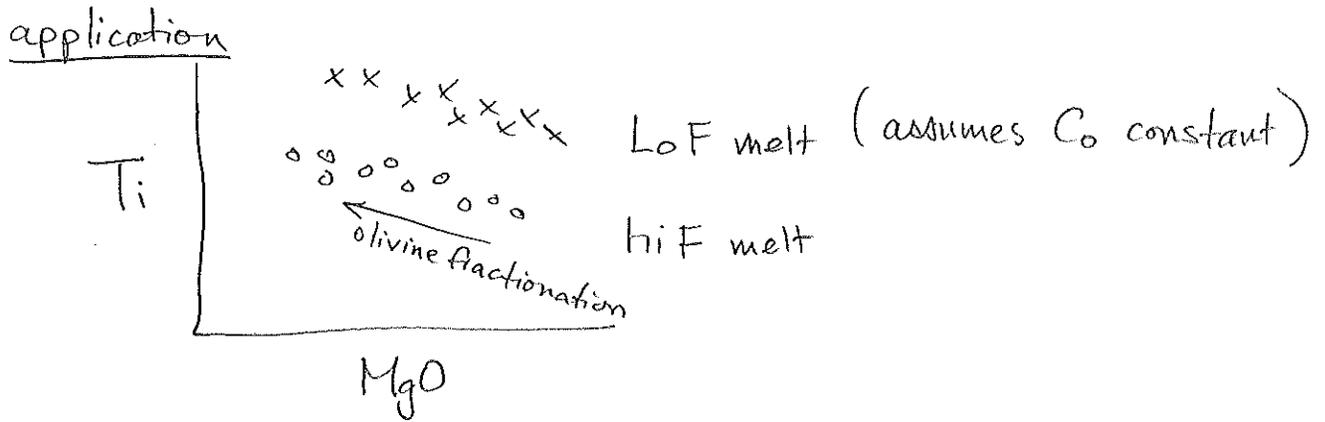
TEMPERATURE — THERMOMETER

T | olivine (Fo ⟶ Fa curve)

Fo   X   Fa
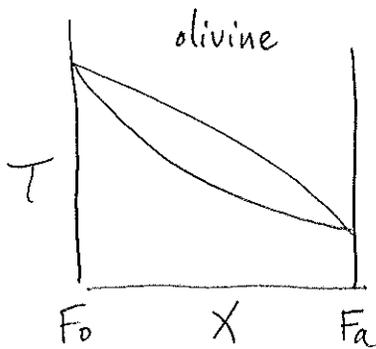
$$k_d = \frac{MgO_{olivine}}{MgO_{Liquid}}$$

$$\ln k_d = \frac{a}{T} + b$$

$\ln k_d$ | (increasing line) $1/T$
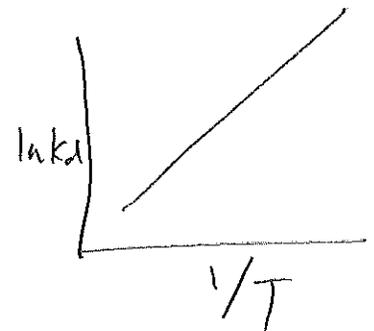
FROM MgO content of melt, you can get T

✱ High MgO ⟶ hi T (as long as olivine saturated)

IN DETAIL

$$Fe_2SiO_{4\,melt} + Mg_2SiO_{4\,olivine} = Fe_2SiO_{4\,oliv} + Mg_2SiO_{4\,melt}$$

$$K_D = \frac{(Fe/Mg)_{ol}}{(Fe/Mg)_m} \sim 0.3 \quad \text{relatively constant with } T, P$$

Fo90   Fo88

Mg mole fract.   ×× ×× ×× ×× ×× ↓ FRACT.   PRIMARY magma ⟶ T

Temp hi

T low   ROEDER + EMSLIE

Fe mole fraction

12

Pressure

Fo | En   X   Qtz

↑
PERIDOTITE
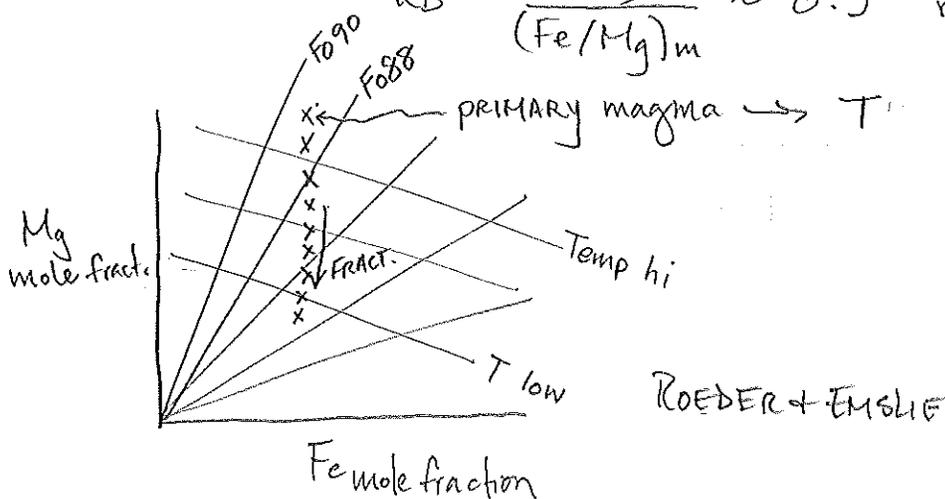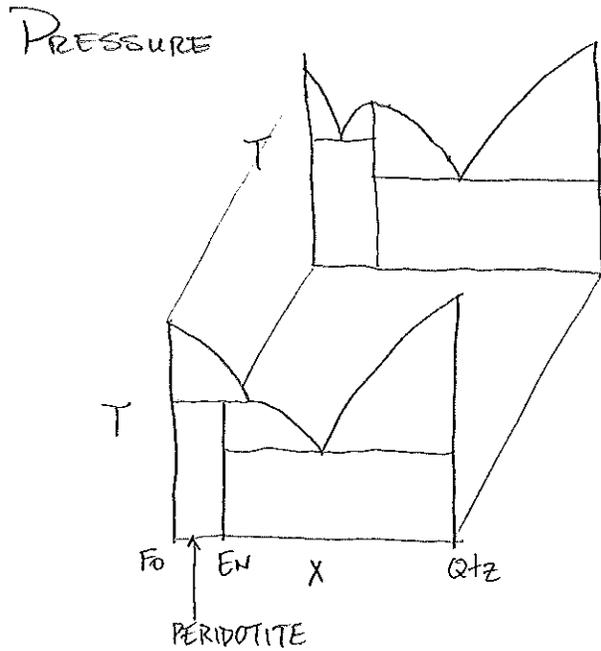
A PRESSURE-SENSITIVE RXN

$$Mg_2SiO_4 + SiO_2 = Mg_2Si_2O_6$$

ol   melt   px
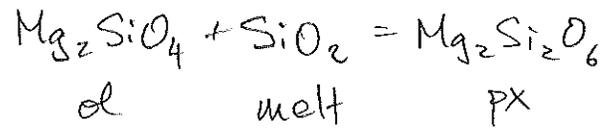
$$K = \frac{a_{px}^{En}}{a_{ol}^{Fo}\, a_{melt}^{SiO_2}}$$

$$\frac{a_{px}^{En}}{a_{ol}^{Fo}} \sim constant$$

$$K \sim \frac{1}{a_{melt}^{SiO_2}}$$

OLIV + OPX SYSTEM
BUFFERS SiO_2

RULES OF THUMB

* Low SiO_2 ⇝ hi P (assuming you have a
primary magma)

CAVEATS/COMPLICATIONS

- MANTLE SOURCE VARIABLE IN TERMS OF MAJOR COMPOSITION
- PRIMARY MAGMA NOT KNOWN
- MAGMAS ARE AGGREGATES OR REACTED PRODUCTS.